



Continuum of earthquake rupture speeds enabled by oblique slip

Huihui Weng and Jean-Paul Ampuero

Earthquake rupture speed can affect ground shaking and therefore seismic hazard. Seismological observations show that large earthquakes span a continuum of rupture speeds, from slower than Rayleigh waves up to P-wave speed, and include speeds that are predicted to be unstable by two-dimensional theory. This discrepancy between observations and theory has not yet been reconciled by a quantitative model. Here we present numerical simulations that show that long ruptures with oblique slip (both strike-slip and dip-slip components) can propagate steadily at various speeds, including those previously suggested to be unstable. The obliqueness of slip and the ratio of fracture energy to static energy release rate primarily control the propagation speed of long ruptures. We find that the effects of these controls on rupture speed can be predicted by extending the three-dimensional theory of fracture mechanics to long ruptures with oblique slip. This model advances our ability to interpret supershear earthquakes, to constrain the energy ratio of faults based on observed rupture speed and rake angle, and to relate the potential rupture speed and size of future earthquakes to the observed slip deficit along faults.

Earthquake rupture speed controls seismic energy radiation and therefore affects ground shaking and seismic hazard, yet the quantitative factors controlling the rupture speed of large earthquakes are still not completely understood and the speeds of some earthquakes remain unreconciled with basic models. In general, faster ruptures can generate stronger ground shaking, near to and far from the fault^{1–4}, unless competing source complexity and path effects are dominant^{5,6}. A compilation of earthquake rupture speeds estimated from seismological observations^{7–14} (Fig. 3a) illustrates that most earthquakes propagate at speeds slower than the S-wave speed, v_s , and some^{11,12} at speeds faster than the Eshelby speed, $v_E = \sqrt{2}v_s$ (hereafter called fast-supershear earthquakes). Recent evidence^{7,8} shows that supershear earthquakes can also propagate steadily at sub-Eshelby speeds (hereafter called slow-supershear earthquakes), which is unexpected according to the two-dimensional (2D) theory of fracture mechanics¹⁵. Such unexpected speeds have been reported in large earthquakes, with ruptures much longer than they are wide^{16,17}. The propagation of ruptures with large aspect ratio has been studied theoretically in three dimensions in mode III, corresponding to pure dip-slip faulting¹⁸. By extending that theory to three-dimensional (3D) mode II ruptures, we found that slow-supershear speeds are also inadmissible for long, steady, pure strike-slip earthquakes (Methods), as in the 2D theory. However, natural earthquakes generally have oblique slip, with both strike-slip and dip-slip components¹⁶. A 2D computational study¹⁹ has suggested that such mixed-mode ruptures can propagate at speeds between the Rayleigh wave speed (v_R) and v_s , which is a forbidden zone for pure mode II rupture. Such speeds have been observed in 3D numerical simulations during only very short transients that would be difficult to observe in nature²⁰. Here, we show that large mixed-mode earthquakes can propagate steadily at speeds spanning the continuum of speeds observed in nature, including forbidden and slow-supershear speeds.

Rupture speed controlled by rake angle and energy ratio

The propagation of long mixed-mode ruptures (Fig. 1) is controlled primarily by two dimensionless quantities, as deduced by

dimensional analysis and confirmed by numerical simulations (Methods): the rake angle (θ) between the initial fault traction and the horizontal direction, and the energy ratio between dissipated and potential energies (G_c/G_0). Here, G_c is the fracture energy dissipated near the rupture front and G_0 is the static energy release rate of mode III subshear ruptures. The latter depends on stress drop and rupture width (W) but not on rupture length, provided rupture length is long compared with W (ref. ¹⁸). A third non-dimensional parameter, the ratio between the size of the weakening process zone at the rupture tip and the rupture width (L_c/W), has a secondary effect on the asymptotic rupture behaviour²¹. Five different rupture behaviours emerge in 3D numerical simulations as the two primary control parameters are systematically varied (Fig. 2a). We first identify two large classes: self-arresting ruptures decelerate and eventually stop spontaneously, while runaway ruptures propagate unabated through the entire fault and eventually approach a steady rupture speed (Fig. 2b). We further classify runaway ruptures according to their steady speed: subshear, forbidden, slow-supershear and fast-supershear ruptures.

Remarkably, long ruptures can propagate steadily at a variety of speeds faster than the Rayleigh wave speed, even at slow-supershear speeds and in the forbidden zone (Fig. 2b). The steady speed of subshear ruptures is v_R for mode II (strike-slip), v_s for mode III (dip-slip) and lies between the two for mixed mode (oblique slip)¹⁹. The steady speeds of supershear ruptures lie between v_s and the P-wave speed (v_p) and decrease as the rake angle and energy ratio increase (Fig. 2c). The same rupture behaviours are identified on the basis of the apparent horizontal speed (Methods), a quantity more usually constrained by seismological analyses, except that apparent horizontal speeds in the forbidden zone are not found (Extended Data Fig. 1).

The conditions separating the different rupture behaviours can be understood and quantitatively predicted by extending the theory of fracture mechanics to 3D mixed-mode long ruptures. A basic element of the theory is that the energy release rate for mixed-mode rupture (G^{mix}) is the sum of the mode II and mode III contributions^{22,23}, and is of the form $G^{\text{mix}} = G_0 f(\theta, v_r)$, where f is a function of

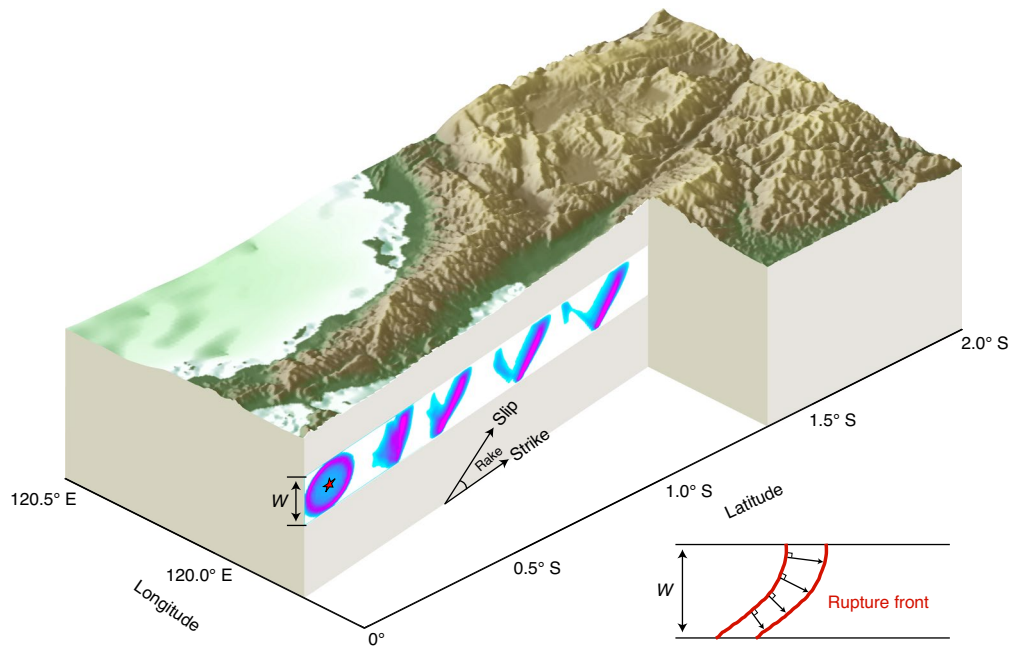


Fig. 1 | Earthquake rupture propagation on a long fault with oblique slip. The fault has a finite width W and slips at an oblique angle (rake) relative to the strike direction. The rupture starts at the hypocenter, indicated by a red star. Colours on the fault plane show the slip velocity at five different times during rupture propagation. The inset shows the propagation of a tilted rupture front on a fault; arrows show the local direction of rupture propagation (real speed vector).

θ and rupture speed (v_r) (Methods). For subshear ruptures, $f(\theta, v_r)$ is independent of v_r . Ruptures are runaway if their energy release rate exceeds the fracture energy, $G^{\text{mix}} > G_c$, otherwise they are self-arresting. Thus the boundary between self-arresting and runaway ruptures satisfies $G_c = G^{\text{mix}}$,

$$G_c/G_0 = f(\theta, v_r) = (1 - \nu)^{-1} \cos^2 \theta + \sin^2 \theta \quad (1)$$

where ν is Poisson's ratio, which is in good agreement with our 3D dynamic simulations' results (Fig. 2a). The values of $f(\theta, v_r)$ at $v_r = v_E$ and $v_r = v_S$ are determined theoretically (Methods) and, in combination with the steady energy balance $G_c = G^{\text{mix}}$, the boundaries between slow- and fast-supershear ruptures and between slow-supershear and forbidden ruptures are well predicted (Fig. 2a).

Ruptures with oblique slip can propagate steadily at slow-supershear and forbidden speeds because their rupture fronts are not vertical but tilted (Fig. 2d and Extended Data Fig. 2b). A kinematic model that captures purely geometric effects, considering an expanding elliptical front with obliquely oriented major axis (Extended Data Fig. 3), qualitatively explains the occurrence of unexpected speeds on long faults but also shows substantial discrepancies with the dynamic model (Methods). Fracture dynamics theory provides a mechanical explanation for the existence of steady rupture speeds in the forbidden zone. While the mode III contribution to the energy release rate is negative in the forbidden zone, in a tilted mixed-mode rupture front it is compensated by the positive mode II contribution (Methods), thus enabling a steady energy balance $G^{\text{mix}} = G_c$.

Model consistency with observed supershear rupture speeds

The theory developed here provides a new interpretive framework for supershear earthquakes that suggests a method to constrain the energy ratio G_c/G_0 of faults based on observations of earthquake rupture speed and rake angle. Model and observations can be compared in terms of rupture speed, rake angle and energy ratio

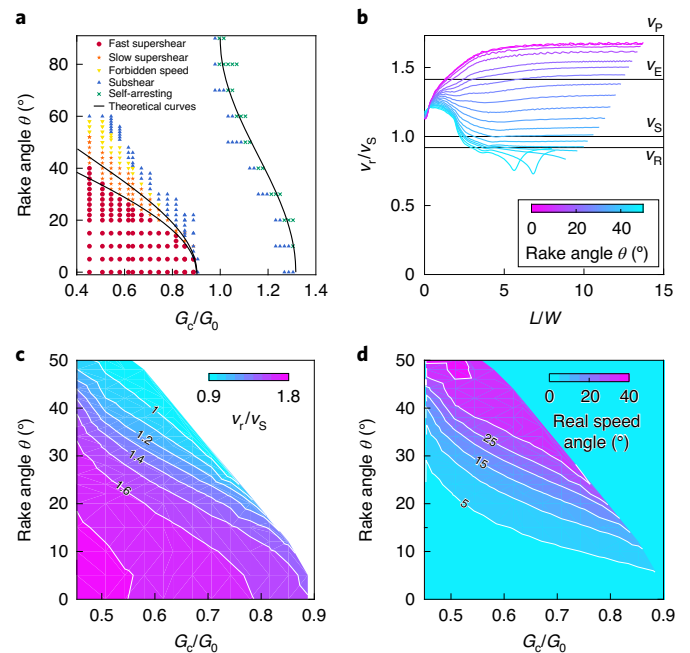


Fig. 2 | Rupture propagation controlled by energy ratio and rake angle.

a, Five different rupture behaviours (legend) found in 3D dynamic rupture simulations depending on energy ratio G_c/G_0 and rake angle θ . Black curves are theoretical estimates of boundaries between certain behaviours. For subshear and self-arresting ruptures, only cases near the theoretical boundary were simulated. **b**, Normalized depth-averaged rupture speed v_r/v_s versus normalized distance L/W , colour coded by rake angle, for $G_c/G_0 = 0.63$. v_R , v_S , v_E , and v_P are the Rayleigh wave, S-wave, Eshelby and P-wave speeds, respectively. **c, d**, Normalized steady rupture speed (**c**) and real speed angle (**d**) as a function of G_c/G_0 and θ . Only fast-supershear, slow-supershear and forbidden speeds (legend in **a**) are shown in **c**.

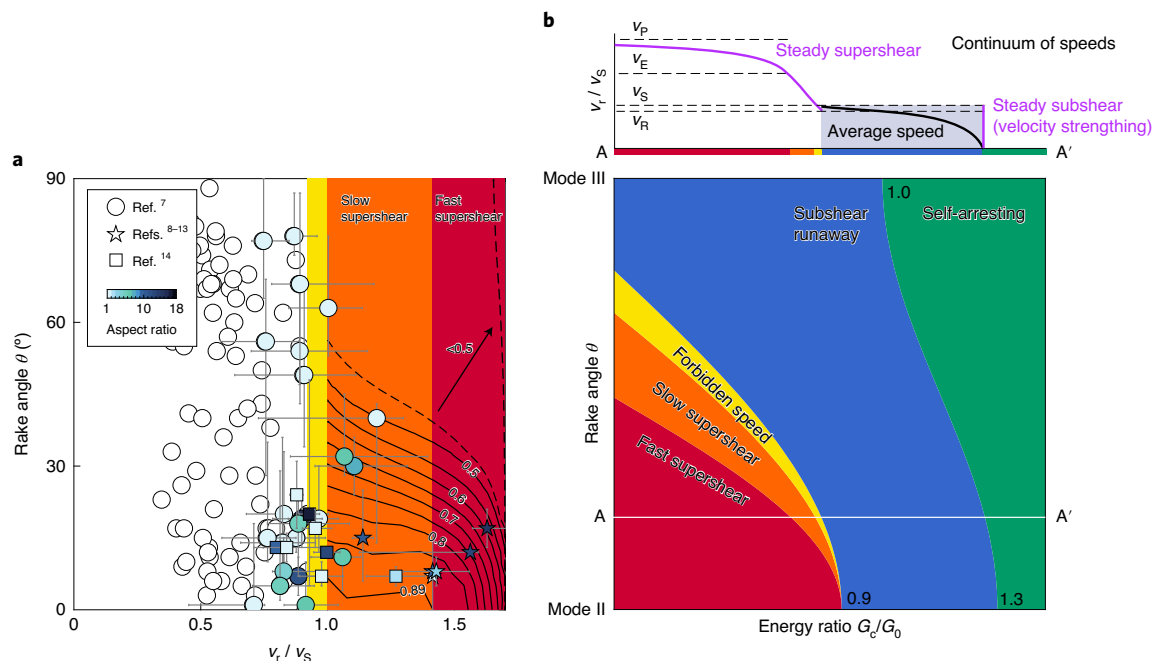


Fig. 3 | Comparison between observed and simulated speeds and synopsis of rupture behaviours. **a**, Symbols represent rupture speeds and rake angles of global earthquakes, colour coded by aspect ratio and compiled from refs. ^{7–14}, Earthquake Source Model Database (SRCMOD)¹⁶ and the United States Geological Survey. Error bars indicate uncertainties when reported in references. Speeds are normalized by v_s values compiled from refs. ^{7–14} and the Preliminary Reference Earth Model (PREM)⁴⁴. Solid and dashed curves represent rupture speeds from simulations ($G_c/G_0 > 0.5$) and a conceptual sketch motivated by theory ($G_c/G_0 < 0.5$; Methods), respectively. **b**, Bottom: summary of the five rupture behaviours. Top: rupture speeds versus G_c/G_0 at a fixed rake (conceptual sketch), along profile AA' in the bottom plot. Purple and black curves represent steady-state and distance-averaged (at a fixed rupture length and initial rupture speed) rupture speeds, respectively. Grey box shows all possible values of averaged speeds.

(Fig. 3a). All the supershear earthquakes observed so far have rake angles lower than 60° and a continuum of rupture speeds up to v_p . The basic model explains these earthquake observations and constrains the energy ratios of faults to lie between 0.5 and 0.89. For energy ratios smaller than 0.5, supershear speeds are, in theory, allowed over a wider range of rake angles (dashed curves in Fig. 3a) but have not been observed in nature. A recent example of slow-supershear rupture is the 2018 moment magnitude (M_w) 7.5 Palu earthquake, which was inferred to propagate steadily at a sub-Eshelby speed $\sim 4.1 \text{ km s}^{-1}$ (refs. ^{8,24}). Considering the rakes constrained by different studies of the Palu earthquake ($\sim 25^\circ$ (ref. ²⁴), $\sim 6\text{--}15^\circ$ (ref. ¹⁷) and $\sim 15\text{--}17^\circ$ from the United States Geological Survey and Global Centroid Moment Tensor Catalog), such slow-supershear rupture requires an energy ratio between 0.75 and 0.85. An alternative interpretation of the unusual speed of this earthquake assumes the presence of a low-velocity fault zone²⁵, which remains to be confirmed by local fault studies. The 2013 M_w 6.7 Okhotsk deep earthquake⁹ and the 1999 M_w 7.5 Turkey Izmit earthquake¹⁰ were estimated to propagate at Eshelby speed. This requires values of rake angle and energy ratio near the boundary between slow- and fast-supershear ruptures. The rake angles of these two events are very close to mode II ruptures^{9,16}. Therefore, if these ruptures have a steady Eshelby speed, their energy ratio should be around 0.89 (Fig. 3a); if their speed is not steady and the rupture comprises both super-Eshelby (fast supershear) and subshear segments, this value is an upper bound on the energy ratio for the fault segments with super-Eshelby speed. An example of fast-supershear rupture is the 2001 M_w 8.1 Kunlun earthquake¹¹. An intermediate portion of the rupture had super-Eshelby speed $\sim 5 \text{ km s}^{-1}$ and rake $\sim 10^\circ$ (ref. ¹⁶), which requires $0.7 < G_c/G_0 < 0.8$.

The model presented here also explains the continuum of earthquake rupture speeds, ranging from slower than Rayleigh wave

speeds to P-wave speed (Fig. 3b). For subshear runaway ruptures, steady propagation at speeds arbitrarily lower than the S-wave speed requires the fracture energy to increase with rupture speed, which can result from velocity-dependent friction¹⁸. Otherwise, subshear runaway ruptures accelerate to a rake-dependent steady speed between v_R and v_s and, for a given rupture length, their average rupture speed increases from 0 to v_s as the energy ratio decreases. In the forbidden, slow-supershear and fast-supershear regimes, ruptures can propagate steadily at speeds between v_R and v_p , even in the absence of velocity-dependent friction: they are stable because the velocity-dependence of energy release rate can stabilize perturbations of rupture speed (Methods).

Towards physics-based seismic hazard assessment

The fracture mechanics theory of long ruptures developed here is an important step towards a physics-based framework that relates the time-dependent seismic hazard along large faults to quantities that can be constrained and monitored by geophysical observations, such as seismic coupling (Fig. 4). Under similar simplifying assumptions as considered here, namely for connected ruptures along planar faults, a rupture potential (Φ) was introduced by Weng and Ampuero¹⁸ to infer the arrest distance of long dip-slip (mode III) ruptures with a given spatial distribution of G_c/G_0 along strike. We heuristically extend their definition to mixed-mode long faults as:

$$\Phi(L_1, L_2) = \int_{L_1}^{L_2} (1 - G_c/G^{\text{mix}}) dL/W \quad (2)$$

where $G^{\text{mix}} = G_0 f(\theta, v_r)$ is the energy release rate for subshear ruptures and W is the rupture width. The rupture potential serves to anticipate the final size of a rupture: a rupture can propagate over the entire fault segment $[L_1, L_2]$ only if $\Phi(L_1, L_2) > 0$, that is, if the

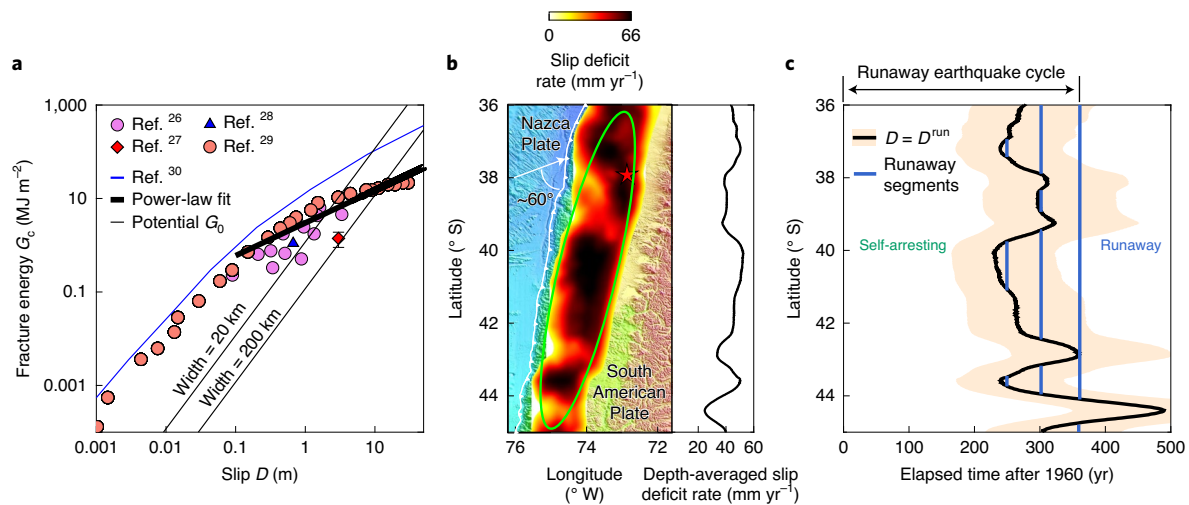


Fig. 4 | Conceptual implications for time-dependent seismic hazard assessment. **a**, Symbols represent empirical estimates of fracture energy G_c versus final slip D compiled from refs. ^{26–30}; error bars indicate uncertainty when available. The black thick line represents power-law fit, the blue curve represents an example of theoretical G_c – D relation³⁰ and the black thin lines represent G_0 – D relations. **b**, Spatial distribution (left) and depth-averaged profile (right) of slip deficit rate (seismic coupling ratio³³ times plate convergence rate (~ 66 mm yr⁻¹)) of the southern Andes subduction zone, Chile; epicentre (red star) and rupture area (green curve) of the 1960 Valdivia earthquake. **c**, Elapsed time since the 1960 earthquake needed to accumulate the critical slip deficit for runaway rupture, D^{run} . The coloured band accounts for uncertainties of the G_c – D scaling relation. Blue lines indicate potentially runaway segments.

average of the mixed-mode energy ratio G_c/G^{mix} along the segment is <1 . In addition, if G_c/G^{mix} is much smaller than 1, such as in the slow-supershear and fast-supershear regimes in Fig. 3b, the rupture of the entire fault segment can be supershear. Therefore, two source properties that can strongly affect seismic hazard, namely rupture length and speed, can be assessed from estimates of the rake angle θ and the energy ratio G_c/G_0 along the fault. The rake angle can be estimated from geodetic data. We propose an approach to estimate the energy ratio at each along-strike location on long faults below.

On the one hand, G_0 on long faults is approximately related to final slip D by $G_0 = C\mu D^2/W$, where C is a geometric factor of order 1 and μ is the shear modulus (Methods). On the other hand, fracture energy G_c can be estimated from scaling relations as a function of final slip D . Such relations have been derived over a wide range of earthquake sizes by different approaches: dynamic earthquake modelling^{26–28}, laboratory experiments²⁹ and seismological methods such as kinematic source inversion^{30,31} (Fig. 4a). As a crude first-order approximation, we seek a scaling relation of the form $G_c \approx BD^n$, where B and n are constant coefficients. Scale-independent G_c corresponds to $n=0$ (for example, linear slip-weakening friction; Extended Data Fig. 4a), while scale-dependent G_c corresponds to $n>0$ (for example, power-law slip-weakening friction; Extended Data Fig. 4b). Theoretical models with off-fault inelastic dissipation^{2,32} lead to $n=1$ and for thermal pressurization³⁰ $n=2/3$. As we focus here on large earthquakes, we only consider the data with $D>0.1$ m and estimate n simply based on the observations. We ignore the data of kinematic source inversions, which probably overestimate the fracture energy due to their over-smoothing of the slip rate function²⁷. Least squares regression gives $n=0.7$ and $B=3$ (the units of G_c and D are MJ m⁻² and m, respectively).

The resulting relation between energy ratio and slip is: $G_c/G_0 = BWD^{n-2}/C\mu$. The spatial distribution of slip deficit rate along a fault can be inferred from geodetic observations^{33–35}. Given an estimate of slip deficit at a future time, a worst-case scenario (largest possible magnitude) is obtained by assuming all the slip deficit is released by a single large earthquake, that is, D is set equal to the slip deficit. Because G_0 depends more strongly than G_c on D ($n<2$), the energy ratio G_c/G_0 decreases with increasing slip deficit D . Thus the condition for runaway ruptures (equation (1)) predicts

that fault segments need to accumulate a certain critical slip deficit $D^{run}(\theta)$ to become capable of hosting long runaway ruptures, otherwise they can only host self-arresting ruptures. Combining the scaling relation of energy ratio versus slip with equation (2) allows us to infer an estimate of the largest possible rupture size from a slip deficit distribution. As an illustration, the time-dependent evolution of the segmentation of the central Andes subduction zone in Chile predicted by the model is shown in Fig. 4, and yields a reasonable estimate of return time of a 1960-like mega-earthquake of ~ 360 yr (250–500 yr, accounting for model uncertainties). This physics-based estimate of runaway recurrence time is consistent with the kinematic estimate of ~ 300 yr based on slip budget³³. Similarly, a minimum slip deficit value $D^{sup}(\theta)$ is required for steady supershear ruptures (Methods). The model also implies that, on a given fault, supershear earthquakes should have larger slip than subshear ones, which is supported by a compilation of observations of previous supershear earthquakes showing that rupture portions that developed supershear speeds tended to host the area of largest slip in each earthquake^{11,36–38}.

Given its simplifying assumptions, the model developed here represents a fundamental building block towards models that are applicable also to ruptures that are strongly affected by complexities such as heterogeneous initial stress and friction properties, non-planar fault geometry, heterogeneous media. Promisingly, previous results^{39–41} have suggested that the model can be extended to account for complexities with length scales smaller than the seismogenic width. The effects of low-velocity damaged zones on 3D shear ruptures can be explained by a reduced effective shear modulus³⁹, heterogeneous fracture energy can be represented by an equivalent homogeneous G_c (ref. ⁴⁰) and fault roughness can be accounted for in G_0 (ref. ⁴¹). However, these topics warrant further investigation in the context of 3D shear ruptures. Moreover, further work is required to account for large-scale complexities found in real fault systems, such as fault segmentation, branching and stepovers^{42,43}.

While there are considerable uncertainties in some model inputs and important fault complexities still need to be accounted for, continued efforts to establish robust scaling relations between fracture energy and slip, to better constrain slip deficit accumulation and to develop models including realistic conditions should allow for

integrating the concepts presented here into earthquake hazard assessment. Capitalizing on advances in geodesy, seismology and rock mechanics, the proposed analysis would allow for partitioning a fault into segments with different potential behaviours in future earthquakes: self arresting or runaway, subshear or supershear. By accounting for the finite width of seismogenic zones and the obliqueness of earthquake slip, our findings quantitatively reconcile the observations of earthquake rupture speeds with the basic theory of rupture dynamics while opening new avenues for physics-based seismic hazard assessment.

Online content

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <https://doi.org/10.1038/s41561-020-00654-4>.

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Methods

Dynamic rupture simulations. We set 3D dynamic rupture simulations with oblique slip on a long fault with finite seismogenic width W embedded in an unbounded, linear elastic, homogeneous medium. We use a computational domain large enough to avoid the effects of the reflected waves from the domain boundaries within the simulation time. We assume a Poisson's ratio ν of 0.25. The shear modulus and S-wave speed of the medium are denoted μ and v_s , respectively. The P-wave speed, the Eshelby speed and the Rayleigh wave speed are $v_p = \sqrt{3}v_s$, $v_E = \sqrt{2}v_s$ and $v_R = 0.92v_s$, respectively.

We use the linear slip-weakening friction law with slip-weakening distance d_c , static strength τ_s and dynamic strength τ_d . This is the simplest friction law adopted in computational earthquake dynamics, and allows for prescribing a constant fracture energy $G_c = 0.5d_c(\tau_s - \tau_d)$ (Extended Data Fig. 4a). The strength values are also fixed because the fault normal stress is constant due to the symmetries of the problem. For a pure dip-slip fault (rake angle of 90°), Weng and Ampuero¹⁸ demonstrated that the key parameter that controls the evolution of rupture speed is the energy ratio G_c/G_0^{II} , where the energy release rate is $G_0^{\text{II}} = \lambda_{\text{II}} \Delta\tau^2 W / \mu$, $\Delta\tau = \tau_0 - \tau_d$ is the nominal stress drop, τ_0 is the initial shear stress and λ_{II} is a geometric factor of order 1. The definition of the mode II energy ratio G_c/G_0^{II} is the same⁴⁵ except for the value of the geometric factor λ_{II} . The energy ratio for purely mode II or purely mode III (assuming the same stress drop $\Delta\tau$) can be written as:

$$\frac{G_c}{G_0^{\text{II}}} = \frac{1}{2\lambda_{\text{II}}} \frac{L_c}{W} \left[\frac{\Delta\tau}{\tau_s - \tau_d} \right]^{-2} \quad (3)$$

$$\frac{G_c}{G_0^{\text{III}}} = \frac{1}{2\lambda_{\text{III}}} \frac{L_c}{W} \left[\frac{\Delta\tau}{\tau_s - \tau_d} \right]^{-2},$$

where

$$L_c = \frac{\mu d_c}{\tau_s - \tau_d} \quad (4)$$

is a characteristic frictional length proportional to the static cohesive zone size⁴⁶. The value of λ_{II} was determined analytically and validated numerically¹⁸: $0.96/\pi$ for a deep buried fault (infinite space, like considered here), $1.92/\pi$ for a surface-breaking fault in a half-space, and between $0.96/\pi$ and $1.92/\pi$ for a buried fault in a half-space. Here, we found numerically for mode II ruptures on a deep buried fault that $\lambda_{\text{II}} \approx 0.96/\pi(1-\nu)$, which is similar to the value 0.43 obtained by Weng and Yang⁴⁵. Then we have $\lambda_{\text{II}}/\lambda_{\text{III}} = (1-\nu)^{-1}$. In the main text, we denote $G_0 = G_0^{\text{II}}$, and therefore $G_0^{\text{II}} = (1-\nu)^{-1}G_0$. To prescribe the energy ratio G_c/G_0 , we fix the value of the cohesive ratio $L_c/W = 0.25$ and vary the stress ratio $\Delta\tau/(\tau_s - \tau_d)$. Note that here we denote $\Delta\tau$ as the amplitude of the stress drop vector. The minimum value of the energy ratio is proportional to the cohesive ratio, $G_c/G_0 \propto L_c/W$, and is obtained when the stress drop $\Delta\tau$ equals the strength drop $\tau_s - \tau_d$ (in such extreme case, the P wave from the hypocentre can trigger the rupture of the entire fault, enabling rupture at the P-wave speed for all mixed-mode ruptures). Since we consider oblique slip with rake angle θ (the direction between the initial traction vector and the horizontal direction), the initial shear stress, whose amplitude is τ_0 , has an along-strike component $\tau_0 \cos \theta$ and along-dip component $\tau_0 \sin \theta$. Exploiting the symmetries of the problem, we only need to simulate rake angles between 0° and 90° . Other values of θ' between -180° and 180° can be mapped to the $0-90^\circ$ range as $\theta = \min(|\theta'|, 180 - |\theta'|)$. If the absolute initial stress τ_0 is not much larger than the stress drop $\Delta\tau$, the slip direction may be time dependent inside the cohesive zone¹⁹ and thus the actual fracture energy may be larger than G_c . To have full control on the actual value of the fracture energy, we set up a relatively large initial stress, $\tau_0/\Delta\tau \approx 10$.

We prescribe a time-dependent weakening over the nucleation zone of size $L/W = 2$ to nucleate unilateral ruptures at prescribed speeds. Rupture propagation becomes spontaneous outside the nucleation zone. To study steady supershear ruptures, without focusing on the supershear transition, we set the nucleation speed as $1.1v_s$ or $1.414v_s$. Tests show that the value of the nucleation speed does not affect the steady-state supershear speed (Extended Data Fig. 5). To study self-arresting and runaway ruptures, we use a sub-Rayleigh nucleation speed of $0.5v_s$.

We use the spectral element software SPECfEM3D⁴⁷⁻⁵⁰ for the dynamic simulation. All the simulations are conducted on a medium-scale computing cluster with 64 cores and 384 GB memory. We set the time step based on the Courant–Friedrichs–Lewy stability condition. To guarantee sufficient numerical resolution, we set a grid size (Δx) much smaller than the characteristic frictional length, that is, $L_c/\Delta x = 10$. We also test a few models with refined grid, $L_c/\Delta x = 20$, and find their results are the same.

Calculations of rupture speed. We compute two types of rupture speed: depth-averaged real speed v_r and apparent horizontal speed v_r^{hor} (Extended Data Fig. 2a). First, the real speed is computed at each point on the fault from the gradient of rupture time $t(x_1, x_3)$

$$v_r(x_1, x_3) = \frac{1}{\sqrt{(\partial t/\partial x_1)^2 + (\partial t/\partial x_3)^2}} \quad (5)$$

where x_1 and x_3 are the along-strike and along-depth distances, respectively. The horizontal apparent speed is based on the horizontal gradient of rupture time:

$$v_r^{\text{hor}}(x_1, x_3) = \frac{1}{\partial t/\partial x_1}. \quad (6)$$

Then, we averaged the real speed and apparent horizontal speed along depth at each along-strike position.

Energy release rate for mode II supershear rupture. The 2D theory predicts the energy release rate of supershear ruptures has the following form⁵¹:

$$G = g(v_r) \frac{\Delta\tau^2 L}{\mu} \left(\frac{\Lambda}{L} \right)^{p(v_r)} \quad (7)$$

where $g(v_r)$ and $p(v_r)$ are known functions of rupture speed, L is the rupture propagation distance, and Λ is the size of the dynamic cohesive zone, $\Lambda \propto L_c$. In general, $g(v_r)$ depends on the shape of the slip-weakening curve⁵¹, but in this study the friction law is fixed. In 2D, G increases from 0 at $v_r = v_s$ to its peak value at $v_r = v_E$, and then decreases to 0 at $v_r = v_p$. As $p(v_r) < 1$ for all speeds between v_s and v_p , $G \propto L^{1-p(v_r)}$ is a monotonously increasing function of L . Hence, for a constant fracture energy G_c , the rupture speed v_r approaches the P-wave speed as L grows. Only if the fracture energy is scale-dependent in the form $G_c \propto L^{1-p(v_r)}$ can steady supershear ruptures exist. Otherwise the only admissible steady speed is the P-wave speed. For elongated ruptures in 3D, the theory by Weng and Ampuero¹⁸ predicts that G saturates when the rupture reaches a finite width W ; it becomes a function of W instead of L . Here, we make heuristic modifications to equation (7) by replacing L with W :

$$G = g(v_r) \left(\frac{\Lambda}{W} \right)^{p(v_r)} G_0. \quad (8)$$

Here $g(v_r)$ differs from the one in the 2D theory by a geometric factor of order 1. The energy balance $G = G_c$ gives

$$\frac{G_c}{G_0} = g(v_r) \left(\frac{\Lambda}{W} \right)^{p(v_r)} \quad (9)$$

We suppose that, like in the 2D case, the right side of the above equation also increases from 0 at $v_r = v_s$ to its peak at $v_r = v_E$, and then decreases to 0 at $v_r = v_p$. This equation of motion of mode II long ruptures predicts that supershear propagation is stable if the energy ratio is below the maximum of the right side of equation (9), which is numerically estimated as $g(v_E) \approx 0.9$ (note that⁵¹ $p(v_E) = 0$). If $G_c/G_0 < 0.9$, there are two mathematical solutions of this equation of motion, one with speed between v_s and v_E and the other between v_E and v_p . Only the latter is stable, because the velocity-decreasing energy release rate provides a negative feedback to any perturbation of rupture speed, which stabilizes steady ruptures. In our 3D, purely mode II dynamic simulations, we only observe steady supershear ruptures at speeds between v_E and v_p (Fig. 2a), which is well explained by the heuristic equation of motion.

Energy release rate for mixed-mode rupture. For mixed-mode ruptures in 3D faults with finite width W , we use a reduced-dimensionality (2.5D) model to derive the energy release rate. The 2.5D model has been previously shown to be a very good approximation of the 3D elongated rupture model for pure dip-slip faulting¹⁸. It assumes that the rupture front is nearly vertical. This assumption is reasonable because in the 3D dynamic simulations, the angles of mixed-mode rupture fronts are quite small ($< 10^\circ$) for fast-supershear, subshear runaway and self-arresting ruptures (Fig. 2d). For slow-supershear and forbidden speeds, the rupture front tilt is substantial and its effects cannot be ignored, as explained below.

The energy release rate is the rate of mechanical energy flow into the rupture tip per unit rupture advance. The stress-drop vector (fault-parallel traction change) is approximately parallel to the slip vector, because we focus on situations with little rake rotation. The total energy release rate for a mixed-mode rupture is the sum of the mode II and III contributions^{22,23}, which are associated with the along-strike $\Delta\tau_{\text{str}} = \Delta\tau \cos \theta$ and along-dip $\Delta\tau_{\text{dip}} = \Delta\tau \sin \theta$ components of stress drop, respectively:

$$G^{\text{mix}} = G^{\text{II}} \cos^2 \theta + G^{\text{III}} \sin^2 \theta \quad (10)$$

where G^{II} and G^{III} denote the energy release rates of purely mode II and III ruptures, respectively, that would prevail if both modes had the same stress drop $\Delta\tau$. Equation (10) stems from the energy release rate of each mode being proportional to the square of the associated stress intensity factor (K)^{22,23} and from the principle of linear superposition of stress intensity factors which, for a straight crack, results in K_{II} depending only on $\Delta\tau_{\text{str}}$ and K_{III} only on $\Delta\tau_{\text{dip}}$. The equation also applies to some curved front situations. The stress intensity factors at any point along a static circular rupture front of radius a are⁵²

$$K_{\text{II}} \propto \Delta\tau \sqrt{a} \cos \omega; \quad K_{\text{III}} \propto \Delta\tau \sqrt{a} \sin \omega \quad (11)$$

where ω is the angle between the slip direction and the local rupture propagation direction. The expressions have a similar form at the major axis tip of an elliptical rupture, which can be set horizontal and $a = W$ to draw an analogy to the 2.5D model.

Based on 2.5D models:

$$G^{\text{II}} = \begin{cases} (1 - \nu)^{-1} G_0, & \text{if } v_r < v_R \\ G_{\text{fb}}^{\text{II}}, & \text{if } v_R < v_r < v_S \\ g(v_r) \left(\frac{\Lambda}{W}\right)^{p(v_r)} G_0, & \text{if } v_S < v_r < v_P \end{cases} \quad (12)$$

and

$$G^{\text{III}} = \begin{cases} G_0, & \text{if } v_r < v_S \\ G_{\text{Sup}}^{\text{III}}, & \text{if } v_r > v_S \end{cases} \quad (13)$$

where $G_0 = \lambda_{\text{III}} \Delta \tau^2 W / \mu$, $G_{\text{fb}}^{\text{II}}$ and $G_{\text{Sup}}^{\text{III}}$ are theoretical energy release rates within forbidden speed ranges, and v_r is the depth-averaged real speed. The 2D analytical solutions of $G_{\text{fb}}^{\text{II}}$ and $G_{\text{Sup}}^{\text{III}}$ depend on different mathematical assumptions: inverse-square-root singularity or no singularity of stress at the rupture tip^{15,19,53}. In the 2.5D model, under the same mathematical assumptions, the singular solution yields the same expressions as for sub-Rayleigh ruptures but with negative values, $G_{\text{fb}}^{\text{II}} = -(1 - \nu)^{-1} G_0$ and $G_{\text{Sup}}^{\text{III}} = -G_0$, and the non-singular solutions $G_{\text{fb}}^{\text{II}} = 0$ and $G_{\text{Sup}}^{\text{III}} = 0$. Within the ranges bracketed by these two end-member analytical solutions, the values of $G_{\text{fb}}^{\text{II}}$ and $G_{\text{Sup}}^{\text{III}}$ appropriate to describe our simulations are determined empirically as explained below.

The mixed-mode energy release rate is of the form $G^{\text{mix}} = G_0 f(\theta, v_r)$. Self-arresting ruptures occur if the energy release rate of mixed-mode steady ruptures is too small to match the fracture energy, $G_c > G^{\text{mix}}$. Runaway ruptures near the boundary with self-arresting ruptures have sub-Rayleigh speeds and almost vertical fronts ($< 5^\circ$; Fig. 2d). Thus, the theoretical boundary between self-arresting and runaway ruptures corresponds to the condition $G_c = G^{\text{mix}}$ evaluated at sub-Rayleigh speeds ($v_r < v_R$):

$$f(\theta, v_r) = G_c / G_0 = (1 - \nu)^{-1} \cos^2 \theta + \sin^2 \theta. \quad (14)$$

Note that at sub-Rayleigh speeds $f(\theta, v_r)$ does not depend on v_r . The theoretical boundary between fast-supershear and slow-supershear ruptures is obtained by evaluating the energy balance $G_c = G^{\text{mix}}$ at $v_r = v_E$:

$$G_c = g(v_E) G_0 \cos^2 \theta + G_{\text{Sup}}^{\text{III}} \sin^2 \theta \quad (15)$$

where $\left(\frac{\Lambda}{W}\right)^{p(v_E)} = 1$ because $p(v_E) = 0$ and we know that $g(v_E) = 0.9$. We find that if we set an empirical value of $G_{\text{Sup}}^{\text{III}} / G_0 = -0.4$ (amid the two end-member analytical solutions) in the theoretical equation (15) the resulting equation fits the Eshelby boundary from our 3D dynamic simulations well (Fig. 2a):

$$f(\theta, v_E) = G_c / G_0 = 0.9 \cos^2 \theta - 0.4 \sin^2 \theta. \quad (16)$$

For the boundary between supershear and subshear regimes, the theoretical relation is

$$G_c = g(v_{S+}) \left(\frac{\Lambda}{W}\right)^{p(v_{S+})} G_0 \cos^2 \theta + G_{\text{Sup}}^{\text{III}} \sin^2 \theta \quad (17)$$

where v_{S+} is a rupture speed slightly larger than the S-wave speed. Near this boundary, the rupture front tilts severely, $\sim 30^\circ$ (Fig. 2d), therefore the effects of tilted rupture front needs to be considered. The first term on the right side of equation (17) is positive and the second term is non positive. The first term on the right side of equation (17) need to be sufficient to support the dissipated fracture energy. As the term $g(v_{S+})$ is quite small according to the theory, we suppose that the geometric effect of tilted front enlarges the size of the apparent cohesive zone along the strike direction to make the first term sufficiently large. We find that if we set empirical values of $g(v_{S+}) \left(\frac{\Lambda}{W}\right)^{p(v_{S+})} = 0.9$ (same as equation (16)) and $G_{\text{Sup}}^{\text{III}} / G_0 = 0.0$ (one end-member analytical solution) in the theoretical equation (17) the resulting equation fits the supershear boundary for small rake angle well (Fig. 2a):

$$f(\theta, v_S) = G_c / G_0 = 0.9 \cos^2 \theta. \quad (18)$$

For larger rake angle, equation (18) underestimates the energy release rate due to the even larger tilt of the rupture front ($> 30^\circ$, Fig. 2d).

Scaling relation of energy release rate. On long faults, the depth-averaged static energy release $G_0(x)$ is related to final slip $D(x, z)$ ⁵⁴ by

$$G_0(x) = \frac{1}{2} \int_0^W \Delta \tau(x, z) D(x, z) dz / W \quad (19)$$

where x and z are along-strike and along-dip distances, respectively. To first order, $G_0(x) = \frac{1}{2} \Delta \tau(x) D(x)$ (Extended Data Fig. 4) and $\Delta \tau(x) = 2\mu D(x) / W$, therefore this equation is approximated as

$$G_0(x) = \frac{C\mu D(x)^2}{W} = \frac{1}{4C} \frac{\Delta \tau(x)^2 W}{\mu} \quad (20)$$

where C is a geometric factor of order 1 and $\Delta \tau(x)$ and $D(x)$ are the depth-averaged stress drop and slip, respectively. We retrieve $G_0(x) = G_0^{\text{III}}(x)$ by taking $C = 1/(4\lambda_{\text{III}})$ for mode III and $G_0(x) = G_0^{\text{II}}(x)$ by taking $C = 1/(4\lambda_{\text{II}})$ for mode II, respectively. These two values of C agree within 4% with previous analytical solutions of static energies in full space, that is, $C = 2/\pi/(1 - \nu)$ for mode III⁵⁵ and $C = 2/\pi$ for mode II^{56,57}, respectively. For a mixed-mode rupture,

$$\begin{aligned} \Delta \tau^2 &= \Delta \tau_{\text{str}}^2 + \Delta \tau_{\text{dip}}^2 \\ \Delta \tau_{\text{str}} &= \frac{(1 - \nu)\pi\mu}{2W} D_{\text{str}} \\ \Delta \tau_{\text{dip}} &= \frac{\pi\mu}{2W} D_{\text{dip}} \end{aligned} \quad (21)$$

where D_{str} and D_{dip} are along-strike and along-dip slip components. Since we focus on situations with little rake rotation, we have

$$\begin{aligned} D_{\text{str}} &= D \cos \theta \\ D_{\text{dip}} &= D \sin \theta. \end{aligned} \quad (22)$$

Therefore, the factor relating stress drop $\Delta \tau$ and final average slip D for mixed-mode rupture is

$$C = \frac{\pi}{4} \sqrt{(1 - \nu)^2 \cos^2 \theta + \sin^2 \theta} \quad (23)$$

For a half-space with rupture breaking the surface, C is divided by two¹⁸. For a buried fault, the value of C lies between the values of the full-space and half-space models. Our conceptual conclusions hold qualitatively for all values of C . The quantitative effect of C on our results is encapsulated in equation (20); in particular, it affects the value of the critical slip deficits for runaway and supershear ruptures, discussed in the main text.

Kinematic model. To investigate the possible reasons for unexpected steady speeds, we compare the dynamic models with a simple kinematic model designed to capture purely geometric effects (Extended Data Fig. 3a). We assume that a supershear rupture extends as an elliptical front propagating at the P-wave speed along its major axis and at the S-wave speed along its minor axis, the limiting speeds for mode II and III ruptures, respectively. The elliptical fronts are truncated to lie inside the seismogenic portion of the fault. The rake angle is the angle between the major axis of the ellipse and the strike direction. We vary the rake angle and compute the depth-averaged real speed, the horizontal speed and the depth-averaged rupture propagation angle (angle of the real rupture speed relative to the horizontal direction).

We find that the basic geometric effects of tilted elliptical front represented in the kinematic model only account for part of the dynamic simulation results. The rupture speeds decrease as the rake angle increases (Extended Data Fig. 3b,c), as in the dynamic models, but there are also important discrepancies between the two models. Beyond a rake of 20 – 30° , the speed of the dynamic models decreases faster than that of the kinematic models. An eventual drop to sub-Rayleigh speeds is only found in the dynamic models. Furthermore, the variability of the real speed across the depth profile is larger in the dynamic model than in the kinematic model. The dependency of the rupture propagation angle as a function of rake angle (Extended Data Fig. 3d) is totally different between the two models. Also, the variability across depth of the rupture angle is much larger in the dynamic model than in the kinematic model, which means the curvature of the dynamic front is larger than that of the kinematic elliptical front. The rupture angle drops to less than 10° once the speed drops below the Rayleigh speed, only in the dynamic models.

Data availability

The numerical data used for the figures in the main body and in the Extended Data are presented in the Source data. The theoretical data are presented in the Methods. Other data are previously published and available in the references explained in the figure captions. Source data are provided with this paper.

Code availability

The open-source software SPECSEM3D used in our 3D dynamic rupture simulations is available from the Computational Infrastructure for Geodynamics at <https://geodynamics.org/cig/software/specsem3d/>.

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Author contributions

H.W. designed and carried out the numerical experiments, and analysed the numerical results. H.W. and J.-P.A. developed the theoretical model, interpreted the results, and wrote the paper.

Competing interests

The authors declare no competing interests.

Additional information

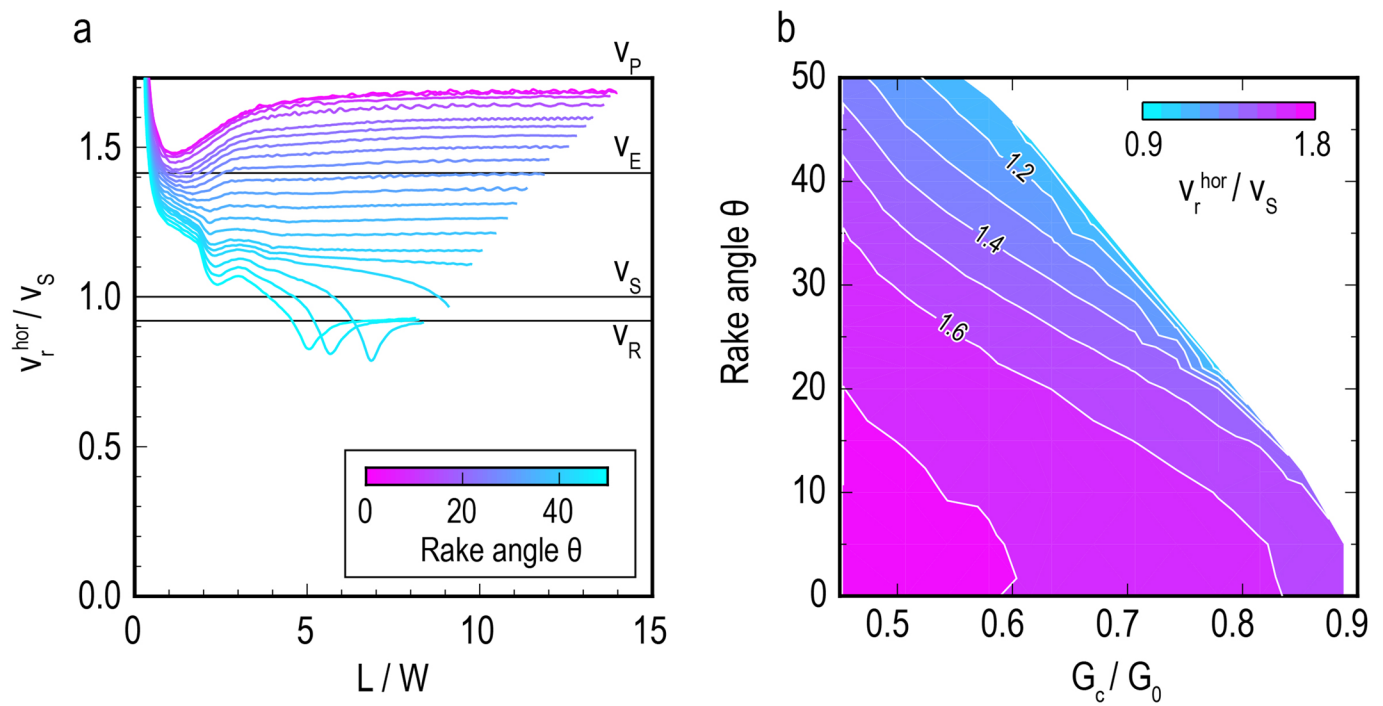
Extended data is available for this paper at <https://doi.org/10.1038/s41561-020-00654-4>.

Supplementary information is available for this paper at <https://doi.org/10.1038/s41561-020-00654-4>.

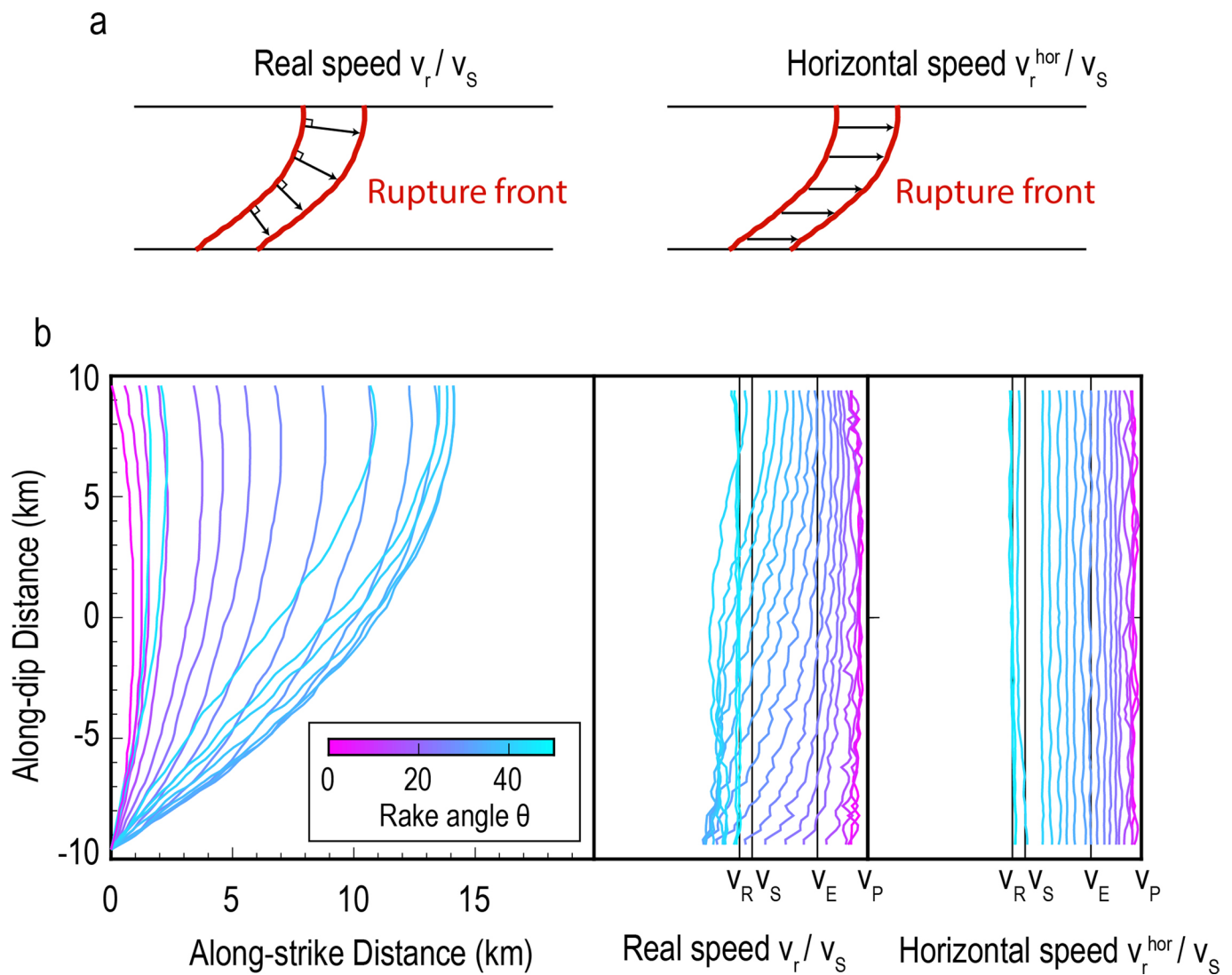
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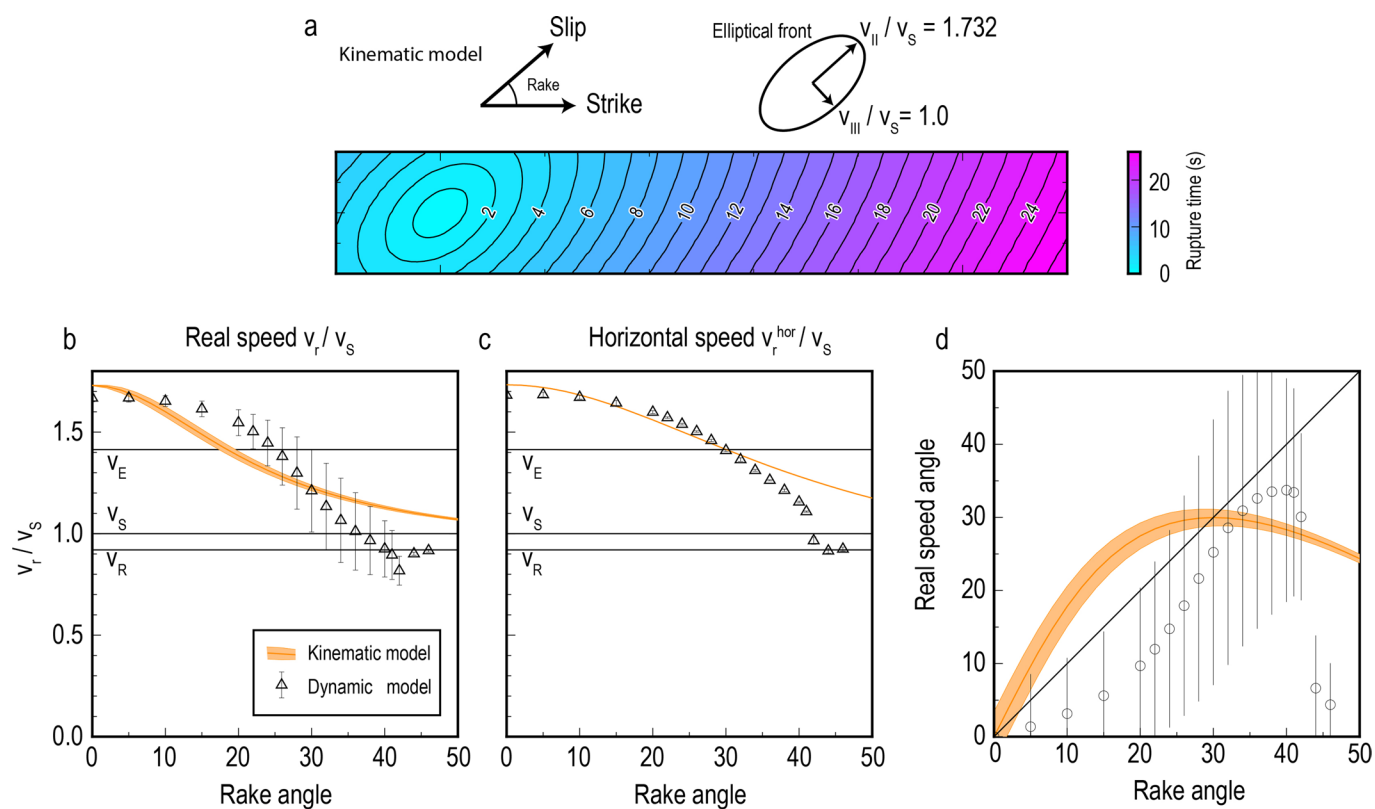
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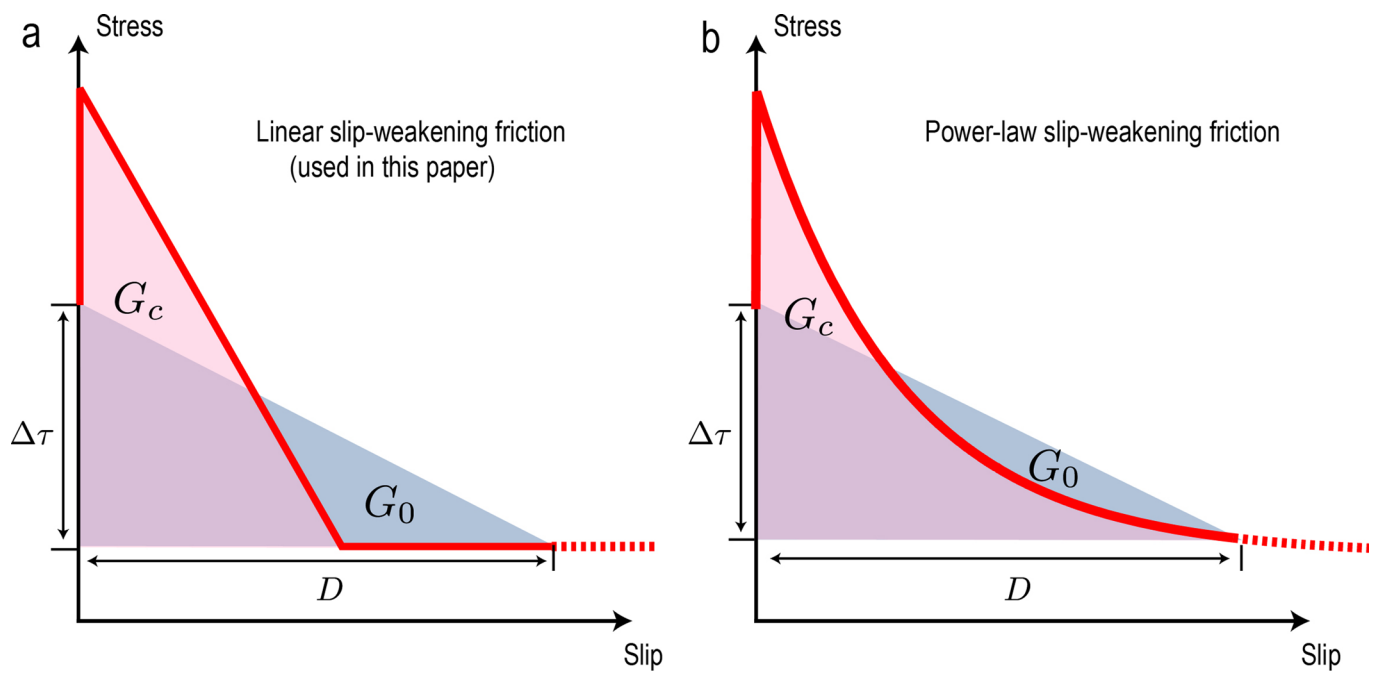
Extended Data Fig. 1 | Horizontal rupture speed controlled by energy ratio and rake angle. **a**, Normalized depth-averaged horizontal speed v_r^{hor} as a function of normalized rupture propagation distance L/W from the 3D dynamic rupture simulations with $G_c/G_0 = 0.63$ and various rake angles (indicated by colors). **b**, Dependencies of normalized depth-averaged horizontal speed on energy ratio and rake angle.



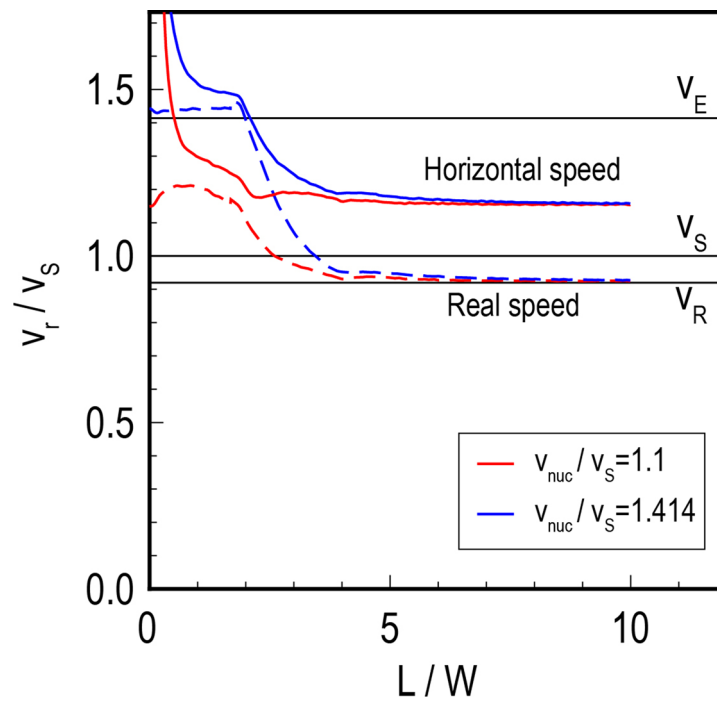
Extended Data Fig. 2 | Rupture speed across depth. **a**, Definitions of real speed (left) and apparent horizontal speed (right). **b**, Shape (left), real speed (middle) and horizontal speed (right) of steady rupture fronts as a function of depth. Colors indicate rake angle.



Extended Data Fig. 3 | Rupture properties of a kinematic rupture model with oblique slip. **a**, Rupture contours of the kinematic model, extending as an elliptical front propagating at the P wave speed along its major axis and at the S wave speed along its minor axis. The rake angle is the angle between the major axis of the ellipse and the strike direction. Comparison between the kinematic and dynamic models with $G_c/G_0 = 0.63$ of **(b)** depth-averaged real speed, **(c)** horizontal speed and **(d)** depth-averaged real speed angle as a function of rake angle.



Extended Data Fig. 4 | Sketches of friction laws. Sketches of fracture energy G_c and static energy release rate G_0 for (a) linear and (b) power-law slip-weakening friction laws.



Extended Data Fig. 5 | Effects of nucleation speed on rupture evolution. Normalized depth-averaged rupture speeds as a function of normalized distance L/W for two 3D dynamic rupture simulations with different nucleation speeds (v_{nuc}).